Vacuum Windows in Recycler Transfer Lines

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For some time now beam transfers between the Main Injector and the Recycler synchrotrons have been plagued by beam loss and fast emittance dilution, with evidence of an amplitude function mismatch between the accelerators presumably generated in the transfer lines. However, there exists in each transfer line a Titanium vacuum window. The effects on emittance and subsequent second-moment oscillation amplitudes seen in the Recycler due to passing through these windows will be described below.

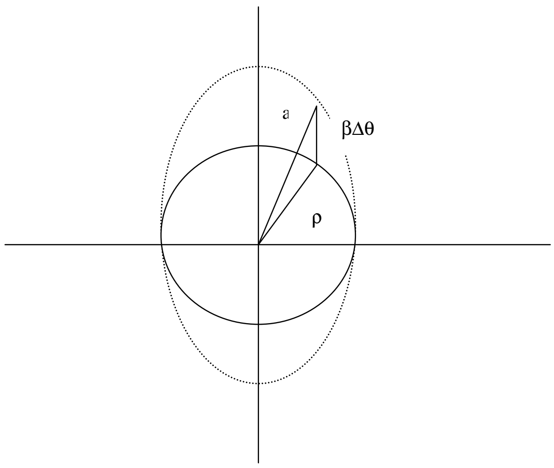

Figure 1: A particle initially at amplitude ρ will acquire a new oscillation amplitude a after being scattered by the vacuum window. The horizontal axis is transverse displacement, x, and the vertical axis is $(\beta x + \alpha x')$.

We assume an otherwise perfectly matched beam between the two accelerators, and look at the effect of the particles passing through a vacuum "window." The process is depicted in Figure 1. The phase space of the beam at the vacuum window is initially circular (in x, $(\beta x' + \alpha x$ coordinates). The particles are following trajectories each with radius ρ_i, and will have an rms transverse amplitude

$\sigma_0 = \langle x^2 \rangle^{\frac{1}{2}} = \langle \rho^2/2 \rangle^{\frac{1}{2}}$. Upon passing through the window, the $i^{\rm th}$ particle's amplitude will change from ρ_i to a_i according to

$$a_i^2 = \rho_i^2 + (\beta\ \Delta\theta_i)^2 - 2\beta\rho_i\Delta\theta_i \cos\phi_i$$

where β is the amplitude function at the window, and $\Delta\theta$ is the scattering angle through the material due to multiple Coulomb interactions. The phase angle, ϕ, depends upon the phase of the particle in its phase space oscillation, and is therefore randomly distributed amongst all the particles, and is uncorrelated with the scattering angle. Therefore, after averaging over all the particles in the beam, we have

$$\langle a^2 \rangle = \langle \rho^2 \rangle + \beta^2 \langle \Delta\theta^2 \rangle - 2\beta \langle \Delta\theta\ \rho \cos\phi \rangle = \langle \rho^2 \rangle + \beta^2 \langle \Delta\theta^2 \rangle\ \ .$$

As depicted in Figure 2, this resulting phase space distribution will tumble through the lattice of the beamline and through the downstream machine, resembling an amplitude function mismatch in that a transverse beam profile monitor will see a "quadrupole oscillation" ensue at twice the betatron tune until the coherent oscillation amplitude dies away due to filamentation.

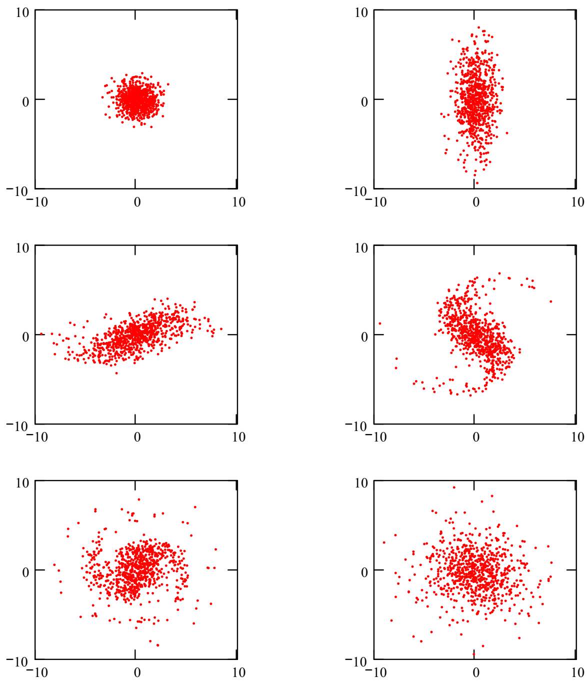

Figure 2: From top left to bottom right: phase space distributions (a) before scattering through the window, (b) immediately after scattering through the window, (c)-(e) during filamentation, and (f) after filamentation is complete. For each plot the horizontal axis is transverse displacement (x), and the vertical axis is $(\beta x + \alpha x')$. The scales are arbitrary units.

Since each particle will continue circulating the accelerator with the betatron amplitude (i.e., single particle invariant emittance) it had acquired just after the window, the resulting phase space in the synchrotron will have a distribution – after filamentation – represented by

$$\sigma^2 = \langle x^2 \rangle_{final} = \langle a^2 \rangle/2 = \langle \rho^2 \rangle/2 + \beta^2 \langle \Delta\theta^2 \rangle/2$$

or

$$\sigma^2 = \sigma_0^2 + \frac{1}{2}\beta^2\theta_{rms}^2 \ .$$

Here, θ_{rms} is the projected rms scattering angle due to multiple Coulomb interactions through the material and can be estimated by[1]

$$\theta_{rms} = \frac{13.6MeV}{pv}\sqrt{\frac{\ell}{L_{rad}}}\,[1 + 0.038\ln(\ell/L_{rad})] \ .$$

This gives a final emittance growth of

$$\Delta\epsilon_N = \frac{6\pi \ \Delta\sigma^2}{\beta}(\gamma\beta) = 3\pi\beta\theta_{rms}^2 \ (\gamma\beta)$$

where $\gamma\beta$ is the Lorentz factor. (Similar treatments of emittance growth from can be found in [2], [3], and also agree with other derivations [4].)

As the beam is injected into the synchrotron the distribution will tumble in phase space such that a profile monitor in the synchrotron will see the beam width vary from turn to turn between values of σ_{min} and σ_{max} with

$$\frac{\sigma_{max}}{\sigma_{min}} = \frac{\sqrt{\sigma_0^2 + \beta^2\theta_{rms}^2}}{\sigma_0} \quad (1)$$

$$= \sqrt{1 + \left(\frac{\beta\theta_{rms}}{\sigma_0}\right)^2} \ . \quad (2)$$

For the case of the Recycler transfer line vacuum windows, we perform the following numerical example. If the amplitude function at the window has a value of 50 m, and the incoming beam has an initial emittance of 10π mm-mrad, then its rms beam size at the window will be about 3 mm. For an 8 GeV (kinetic energy) beam passing through a 5 mil Titanium window ($L_{rad} =$ 3.56 cm) the rms scattering angle is about 72 μrad. This will generate a beam which is mismatched to the optics and which will tumble through the remaining beam line and synchrotron. In the synchrotron, the tumbling will be seen at twice the betatron frequency, and the recorded rms beam size would initially oscillate between maximum and minimum values with ratio $\sigma_{max}/\sigma_{min} \approx 1.6$ until the signal decoheres due to the filamentation. After tumbling (and barring any scraping on apertures, etc.), the resulting emittance increase of the beam would be

$$\Delta\epsilon_N \approx 7\pi \ \ \mathrm{mm-mrad.}$$

One could, in principle, design an optical system which would "match" to the beam distribution downstream of the vacuum window and on into the downstream synchrotron. This would alleviate the problems of mismatch and filamentation in the synchrotron. However, the window would still generate an increase in emittance. If the new distribution could be captured efficiently by an optically matched system, then the emittance of the distribution immediately downstream of the window would be characterized by a phase space area given by

$$\sigma_0\sqrt{\sigma_0^2 + (\beta\theta_{rms})^2}$$

while the original emittance is characterized by an area of σ_0^2. Thus, even if the beam could be optically "captured" immediately after the window its emittance would increase by a factor of

$$\sqrt{1 + \left(\frac{\beta\theta_{rms}}{\sigma_0}\right)^2}$$

which, for the parameters above, would lead to a 50% emittance growth as opposed to a 70% emittance growth if not matched immediately.

For comparison, we note that a 2 mil Titanium window at the same location in the transfer line would generate an rms scattering angle of 44 μrad. The emittance would increase by about 20% immediately after the window. Once injected into the synchrotron the beam would have an rms size that initially oscillates with ratio $\sigma_{max}/\sigma_{min} = 1.2$ and if allowed to filament in the synchrotron it would increase by about 2.7 π mm-mrad.

References

[1] See, for example, Phys. Rev. D, Vol. 66, *Review of particle physics*, July 2002, p. 198

[2] D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, J. Wiley, & Sons, Inc., New York (1993).

[3] P. J. Bryant, *Introduction to transfer lines and circular machines*, CERN-84-04.

[4] C. Johnstone and K. Paul, *Emittance growth from window scattering*, in preparation.